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SPECIAL ISSUE ARTICLE

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Market games with asymmetric information: the core

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Abstract This paper concerns cores of economies with asymmetric information. Alternative definitions of the information available to traders in coalitions and the cooperative games they generate are analyzed. An important technical result states that such NTU games in characteristic function form are well defined. Properties of various cores with asymmetric information are examined. Sufficient conditions on information sharing rules are provided for the induced games to be totally balanced or balanced, so that their cores are nonempty. Incentive compatibility issues are considered. Finally, a perspective on this research area is provided.

Keywords Core · Asymmetric information · Exchange economy · Balancedness

JEL Classification Numbers D51 · D82 · C71

When I first met Mukul Majumdar, he discussed his results on random economies and I have always greatly admired that research. Hence, for this tribute to him, I wanted to contribute a paper on economies under uncertainty.
This is a revised, updated, and shortened version of CARESS Working Paper #91-09, entitled "Market Games with Asymmetric Information and Nontransferable Utility: Representation Results and the Core," which was originally distributed in April 1991. Proofs that do not appear in this paper can be found in Allen (1991a). The research was supported by NSF grant SES88-21442. My interest in the core with asymmetric information was stimulated by fall 1990 Economics 712 advanced topics class at Penn; I would like to thank these graduate students for their patience. John Hillas and Aki Matsui pointed out some embarrassing errors in the notation and definitions used in an early draft presented in winter 1991. The paper was revised during my visit to G.R.E.Q.E. as Invited Professor, Université Aix-Marseille III and later at the University of Minnesota, partially supported by NSF grants DMI-0217974 and DMI-0540440.

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1 Introduction

This paper focuses on the issue of how to define the information that may be used by a coalition of asymmetrically informed members. A key insight is to formalize the information available to coalitions and to examine the games generated by various information specifications systematically. Recent applications of game theory to economics have emphasized noncooperative games rather than cooperative games. Perhaps this can be attributed to our apparent inability to incorporate informational issues into cooperative game theory. This paper makes a contribution in this direction by showing how the asymmetric information in an economy influences the induced cooperative games. The nature of these games with asymmetric information lends insight to the search for reasonable criteria for information sharing conventions for coalitions. [Some previous work has introduced information into cooperative game theory by following the noncooperative games with incomplete information paradigm (see Myerson 1984; Rosenmüller 1990).]

My motivation is derived from microeconomic theory. On a purely theoretical level, a really convincing definition of the core of an economy with asymmetric information has never been achieved. More substantive concerns arise from recent research trends in theoretical industrial organization (information sharing among firms in an oligopolistic industry, cooperative R & D endeavors, and research joint ventures). Cooperative game models with asymmetric information are needed for some of these issues, in that players can make enforceable agreements before they learn the state of the world.

Thus, players' strategies consist of state-dependent net trades to which agents commit before the uncertainty is resolved. Any core payoff vector can be achieved, on average, from feasible net trades that depend only on the information held by traders in the coalition defined by the entire economy; blocking is accomplished through net trades that are feasible for the blocking coalition and depend only on information available to traders in the coalition while providing greater *ex ante* expected utility to coalition members. In this sense, I study a core concept which features *ex ante* blocking based on *ex ante* expected utility payoffs from state-dependent and informationally constrained allocations. As usual, the core should be interpreted as a set to which reasonable outcomes belong in the sense that not every core allocation is economically attractive but points not in the core can be disregarded as somewhat inappropriate.

The obvious reference in this area is the seminal article of Wilson (1978) which proposes definitions of the core (and also for the efficient allocations) of a finite exchange economy with asymmetric information and finitely many states of the world. Wilson discusses the notion of communication structures and focuses on two extreme cases: no use of asymmetric information (termed the null coalition structure) within a coalition, which leads to a core concept in which blocking coalitions can use only that information which is common to all their members, and complete information sharing (the full communication structure), which gives rise to a core in which blocking can be based on any information held by one or more members of the coalition. The latter core may be empty, since blocking is "easy," while the first core is claimed to be large because blocking is "difficult."[1]

[1] Wilson (1968) concerns efficiency for "syndicates," defined as groups of decision makers under uncertainty who share payoffs; risk aversion but not asymmetric information consider-

Using Banach lattice methods, Yannelis (1991) obtained a core existence result, also under the assumption that every event occurs with strictly positive probability so that the set of states of the world is essentially at most countable. The Yannelis core concept is characterized by each trader using private information only. Specifically, the blocking allocation of an individual must be measurable with respect to the individual's initial information, so that joining a coalition has no direct informational consequences. The same information measurability constraint applies to all feasible allocations. Nonemptiness of the private information sharing core arises here as a simple corollary of my approach (see Remark 7.3).

One can speculate that the previous work[2,3] has been concerned almost exclusively with the core as a solution concept for economies with asymmetric information because (1) the core is based on an attractive economic story, (2) it tends to be technically well behaved compared to other alternatives, and (3) it traditionally has been applied to economic models, so that its advantages and disadvantages are fairly well understood. In fact, other cooperative solution concepts have not been modified so as to be defined for economies under uncertainty.

At this point, I also take the core as a convenient starting point cooperative solution concept for the study of coalitions with asymmetric initial information. Specifically, I study the α-core in the sense of Aumann (1961) for nontransferable utility games without side payments. I obtain necessary and sufficient conditions on the information available to coalitions of asymmetrically informed economic agents for all derived market games to be totally balanced or balanced. An implication is the characterization of information sharing rules that give rise to nonempty cores. Related papers analyze the TU core, the core with publicly predictable information, and the value with asymmetric information.

However, I consider some of my more technical results for NTU games with asymmetric information (Section 4) to be probably as important as the core existence and comparison results (primarily contained in Section 7) of this paper. This analysis lays the foundation for the study of cooperative NTU games arising from

ations are included. Kobayashi (1980) relaxed the assumption of finitely many states of the world and proved nonemptiness of the Wilson coarse core (under the assumptions of balancedness and strictly positive probability of every event) using techniques reminiscent of Bewley (1972). He also demonstrated core equivalence for the set of competitive equilibrium allocations in the sense of Radner (1968).

2 Some recent work on optimal taxation has used the core with asymmetric information. For example, Berliant (1991) proves that the fine core analogue of the core without asymmetric information contains only head taxes, while his coarse IC-core may be empty. However, this public finance question involves uncertainty and asymmetric information which is diametrically opposed to the type which interests me. In particular, I am concerned with information about systematic risks, so that there is a single drawing of a state of the world which then becomes an argument of every consumer's utility function. For optimal taxation, idiosyncratic risk is a better description although consumers completely know their own individual drawing of the state of the world. Incomplete information characterizes only the government, which cannot recognize an individual's type and thus cannot necessarily impose type-dependent tax rules. Instead, the government can observe only the distribution of types within a coalition (and not individual identities).

3 Despite its tantalizing title, the article by Muto, Potters, and Tijs (1989) is not relevant in that it concerns transferable utility cooperative games in which any coalition not containing the single "informed" player receives zero; there's no explicit uncertainty. The motivation is patent licensing where the informed player is interpreted as the firm that has already discovered a new technology.

economies with asymmetric information. It should be useful as a first step toward other solution concepts in this framework. In particular, the information sharing rules framework of Section 3 and the demonstration in Section 4 that all models in this class lead to well-defined cooperative games seem especially significant. Notice that closed-valuedness of the correspondence defining NTU games in characteristic function form is technically nontrivial whenever the set of states of the world is (genuinely) uncountable, in that this specification is based on underlying strategies – or information-measurable state-dependent individual net trades in the economic model – that are not contained in a compact set. Note also that my information sharing rules permit arbitrary externalities within any coalition.

Recently a large and growing literature in economic theory has developed on cores with asymmetric information. (Some authors use the term "differential information," but I prefer to avoid this because it has no connection to the notion of differentiability in mathematics.) See, in particular, the survey of incentives and the core of an economy by Forges, Minelli and Vohra (2002), the introduction to the *Economic Theory* special issue by Allen and Yannelis (2001), and the volume edited by Glycopantis and Yannelis (2005).

2 The model

To begin, specify an abstract probability triple $(\Omega, \mathcal{F}, \mu)$ to describe the uncertainty. The set of states of the world is denoted Ω, with typical element ω. Let $\mathcal{F}$ be a σ-field of subsets of Ω, interpreted as the measurable events that economic agents eventually learn, so that events in $\mathcal{F}$ may be payoff relevant for *ex post* utilities. The σ-additive probability measure μ defined on $(\Omega, \mathcal{F})$ represents agents' *ex ante* subjective probabilities attached to the occurrence of various events. To simplify notation, assume that these subjective probability assessments are the same for all agents; this could easily be generalized.

Let I denote the set of economic agents (consumers) in the pure exchange economy. No confusion will result from taking I also to be the set of players in the games we examine. An individual player or trader is denoted by $i \in I$. The set I is assumed to be finite, and we write $\#I$ as its cardinality. Let 2^I denote the set of subsets of I. Nonempty subsets of the player set I are termed coalitions in the game. A *submarket* is a pure exchange economy consisting of only those traders $i \in I'$ for some $I' \subseteq I$, $I' \neq \emptyset$.

Suppose that there is a finite number ℓ of commodities (numbered $1,2,\ldots,\ell$) available in the economy. To summarize endowments, let $e\colon I \times \Omega \to \mathbb{R}^\ell_+$ denote an arbitrary measurable function which is uniformly bounded and write $e_i\colon \Omega \to \mathbb{R}^\ell_+$ for consumer i's random (state dependent) initial allocation function. Define the set E of allocations by $E = \{\, \big(x_1(\cdot), \ldots, x_{\#I}(\cdot)\big) \bigm| \text{for each } i \in I, x_i\colon \Omega \to \mathbb{R}^\ell$ is $\mathcal{F}$-measurable and $-\sum_{i \in I} e_i(\omega) \leq x_i(\omega) \leq \sum_{i \in I} e_i(\omega)$ for almost all $\omega \in \Omega \,\}$.

Consumer i's preferences are specified by a state-dependent cardinal utility function $u_i\colon \mathbb{R}^\ell_+ \times \Omega \to \mathbb{R}$ which is continuous and concave on $\mathbb{R}^\ell_+$ and $\mathcal{F}$-measurable as a function of Ω, so that it is jointly measurable (for the Borel σ-field $\mathcal{B}(\mathbb{R}^\ell_+)$ on i's consumption set $\mathbb{R}^\ell_+$). Assume also that there is some compact convex subset K of $C(\mathbb{R}^\ell_+, \mathbb{R})$ endowed with the (compact-open) topology of uniform convergence on compact subsets of $\mathbb{R}^\ell_+$ such that for (almost) all $\omega \in \Omega$, $u_i(\cdot; \omega) \in K$. This implies that all state-dependent utilities are uniformly equicontinuous and take uniformly (above and below) bounded values on any compact subset of $I\!R^{\ell}_{+}$.

Initial information is represented by sub-σ-fields of $\mathcal{F}$. For $i \in I$, write $\mathcal{G}_i$ for i's initial information. Assume that, for all $i \in I$, $e_i(\cdot)$ is $\mathcal{F}$-measurable and is also known to trader i.

Finally, a trader's goal is to maximize his state-dependent conditional expected utility (which is a $C(I\!R^{\ell}_{+}, I\!R)$-valued random variable – or measurable function – defined on Ω) given his available information. This information can be analyzed by incorporating it into the consumer's objective function (i.e., by calculating conditional expected utilities given the information) as in Allen (1983, 1986a,b). However, a better alternative for the game-theoretic analysis is, whenever possible, to place the information into a measurability constraint on the agent's state-dependent allocation (demand, excess demand, individual net trade, etc.) functions because then the information enters into the definition of commodity spaces but not utilities in our market games. The insight comes from VanZandt (2002). Payoffs to players in our games are taken to equal the expected utilities of final state-dependent commodity allocations. To define conditional expected utilities, we analyze the image measures $\mu \circ u_i^{-1}$ on the Frechet space $C\big(I\!R^{\ell}_{+}, I\!R\big)$ induced by the vector-valued random variables $u_i\colon (\Omega, \mathcal{F}, \mu) \to \big(C(I\!R^{\ell}_{+}, I\!R), \mathcal{B}\big(C(I\!R^{\ell}_{+}, I\!R)\big)\big)$. Then proper versions of regular conditional distributions exist and conditional expected utilities are $C\big(I\!R^{\ell}_{+}, I\!R\big)$-valued random variables that take values (almost surely) in the compact convex set K. In particular, conditional expected utility is (almost surely) continuous and concave on $I\!R^{\ell}_{+}$ (see Rudin 1973, pp. 73–78, for technical details on integration in Frechet spaces).

3 Information sharing within coalitions

To capture the information available within coalitions of asymmetrically informed members in an n-player game, define $2^n - 1$ mappings associating m-tuples (for $0 < m \leq n$) of initial information structures to m-tuples of information structures describing the information that coalition members may use. For notational completeness, I also define the information of the empty set to be the trivial σ-field $\{\Omega, \emptyset\}$. More formally, I define this concept as follows:

Definition 3.1 *An* information sharing rule *is a collection $F = \{ f(S) \mid S \subseteq I \}$ of $2^{\#I}$ mappings for an economy (or game) with asymmetric information. Let $\mathcal{F}^{**}$ denote the set of all sub-σ-fields of $\mathcal{F}$. Then, for $S \subseteq I$, $S \neq \emptyset$, we have*

$$f(S)\colon \underbrace{\mathcal{F}^{**} \times \cdots \times \mathcal{F}^{**}}_{\#\text{S times}} \to \underbrace{\mathcal{F}^{**} \times \cdots \times \mathcal{F}^{**}}_{\#\text{S times}}$$

written as, if $S = \{s(1), \ldots, s(\#S)\}$, $f(S)\big(G_{s(1)}, \ldots, G_{s(\#S)}\big) = \big(H_{s(1)}, \ldots, H_{s(\#S)}\big)$ where, for each $i \in S$, H_i is a sub-σ-field of $\mathcal{F}$. For $S = \emptyset$, we set $f(\emptyset) = \{\Omega, \emptyset\}$. Write $f(S)^i$ for player i's information in coalition S if $i \in S$.

Three concrete examples of information sharing rules are obvious and interesting. However, note that the definition permits all arbitrary possibilities. Morover, there need not be any relation among the information sharing rules used by different coalitions, even those that are subsets or supersets of one another, and the definition does not require that a coalition's information be related to the initial information of its members.

Following the terminology of Wilson (1978), make the following two definitions.

Definition 3.2 *The* coarse information sharing rule *is the (unique) information sharing rule* $F_c = \{ f_c(S) \mid S \subseteq I \}$ *satisfying, for each* $S \neq \emptyset$,

$$f_c(S)\big(\mathcal{G}_{s(1)}, \dots, \mathcal{G}_{s(\#S)}\big) = \Big(\bigcap_{i \in S} \mathcal{G}_i, \dots, \bigcap_{i \in S} \mathcal{G}_i\Big).$$

Definition 3.3 *The* fine information sharing rule *is the (unique) information sharing rule* $F_f = \big\{ f_f(S) \bigm| S \subseteq I \big\}$ *satisfying, for each* $S \neq \emptyset$,

$$f_f(S)\big(\mathcal{G}_{s(1)}, \dots, \mathcal{G}_{s(\#S)}\big) = \Big(\sigma\Big(\bigcup_{i \in S} \mathcal{G}_i, \dots, \sigma \bigcup_{i \in S} \mathcal{G}_i\Big)\Big).$$

Notice that both the coarse information sharing rule and the fine information sharing rule have the property that coalitions are symmetrically informed. For each coalition, it is true that all of its members have exactly the same information available to them regardless of their asymmetric initial information. Say that such an information sharing rule is *symmetric* .

Observe that Definitions 3.2 and 3.3 are related to Wilson's (1978) null and full communication structures. However, they do not lead to precisely the same games analyzed by Wilson (1978) (see Section 10).

To capture the analogous concept used implicitly by Yannelis (1991), we formulate one more definition of a specific information sharing rule by the following:

Definition 3.4 *The* private information sharing rule *is the (unique) information sharing rule* $F_P = \big\{ f_P(S) \bigm| S \subseteq I \big\}$ *satisfying, for each* $S \neq \emptyset$,

$$f_P(S)\big(\mathcal{G}_{s(1)}, \dots, \mathcal{G}_{s(\#S)}\big) = \big(\mathcal{G}_{S(1)}, \dots, \mathcal{G}_{S(\#S)}\big).$$

4 The induced games

Formally, a (cooperative) *nontransferable utility (NTU) game* in characteristic function form is a correspondence $V\colon 2^I \to I\!R^{\#I}$ satisfying $V(\emptyset) = \{0\}$ and, for all $S \subseteq I$, $V(S)$ is nonempty, closed, and comprehensive for $S \neq \emptyset$. Morever, the sets $V(S)$ are "cylinder sets" in that if $(\bar{u}_1, \dots, \bar{u}_{\#I}) \in V(S)$ and $\bar{u}_i = \bar{w}_i$ for all $i \in S$, then $(\bar{w}_1, \dots, \bar{w}_{\#I}) \in V(S)$. Comprehensiveness $\big(V(S) \supseteq V(S) - I\!R^{\#I}_+\big)$ can be interpreted as "free disposability" of utility. Note that I do not require superadditivity.

To derive the NTU game associated with a pure exchange economy with asymmetric information, I must define its characteristic function $V\colon 2^I \to I\!R^{\#I}$ based on the data describing the economy, including its information sharing rule. Accordingly, set $V(\emptyset) = \{0\}$ and for each coalition $S (\emptyset \neq S \subseteq I)$, define $V(S) = \big\{(w_1, \dots, w_{\#I}) \in I\!R^{\#I} \bigm| \text{for } i \in S, \text{ there exist } x_i\colon \Omega \to I\!R^{\ell}_+ \text{ with } w_i \leq \int_\Omega u_i\big(x_i(\omega); \omega\big)\, d\mu(\omega) \text{ such that } z_i(\cdot) = x_i(\cdot) - e_i(\cdot) \text{ is } f(S)^i\text{-measurable and } \sum_{i \in S} x_i(\omega) \leq \sum_{i \in S} e_i(\omega) \text{ for almost all } \omega \in \Omega \big\}$.

Theorem 4.1 *For any information sharing rule, the sets $V(S)$, $\emptyset \subseteq S \subseteq I$, define an NTU game in characteristic function form. In particular, $V(\emptyset) = \{0\}$, and for any $S \neq \emptyset$, $V(S)$ is a nonempty closed comprehensive cylinder set. Moreover, $V(S)$ is convex for all $S \subseteq I$.*

The proof is relegated to the Appendix.

Remark 4.2 Notice that separability of the σ-fields $\mathcal{F}$ and $f(S)^i$ (or of their induced $\mathcal{L}^1$ spaces) was not needed because the $V(S)$ sets are separable as subsets of Euclidean spaces.

Remark 4.3 In a "greatly revised version" [according to the footnote to the title] of a paper that was written after the first version (Allen 1991a) of this paper (see also Allen 1991b, 1992) was circulated, Page (1997) shows that the analysis here can be extended to the case of an underlying infinite-dimensional commodity space; in particular, my state-dependent consumption set $\mathbb{R}^{\ell}_{+}$ can be replaced by the positive core of a Banach lattice. Page (1997) also replaces the assumption that agents' endowments and allocations are bounded almost surely by an integrability condition and generalizes the assumption that state-dependent utilities almost surely lie in some compact convex subset of $C(\mathbb{R}^{\ell}_{+}, \mathbb{R})$ by upper semicontinuity and integrable boundedness. Page (1997) then demonstrates that the resulting NTU game is well defined. Once the NTU game is obtained, his analysis below (that identifies conditions on information sharing that lead to balanced games and thus nonempty cores) follows the same argument.

5 Balancedness

Say that an NTU game[4] $V: 2^I \to \mathbb{R}^{\#I}$ [satisfying $V(\emptyset) = \{0\}$ and for all $S \subseteq I$, the cylinder sets $V(S)$ are closed and comprehensive, where $V(\cdot)$ is a correspondence] is *quasi-balanced* if for every collection $\mathcal{B}$ of subsets S of I and every collection w_S of nonnegative weights for $S \in \mathcal{B}$ with $\sum_{\substack{S \in \mathcal{B} \\ S \ni 1}} w_S = 1$, we have $\bigcap_{S \in \mathcal{B}} V(S) \subseteq V(I)$. It is *totally quasi-balanced* if every subgame is balanced. A *subgame* (the restriction of the original game to a subset of players) of the NTU game $V: 2^I \to \mathbb{R}^{\#I}$ is defined by $V': 2^{I'} \to \mathbb{R}^{\#I'}$ where $I' \subseteq I$ and for every $S \subseteq I$; $V'(S) = \mathrm{proj}_{\mathbb{R}^{\#I'}} V(S)$. To set notation, if $T \subseteq I$, let $V(T)_T$ denote the projection of the set $V(T)$ onto the subspace corresponding to the players who belong to coalition T (or, equivalently, the intersection of $V(T)$ with the subspace $\{z \in \mathbb{R}^{\#I} \mid z_j = 0, \forall j \notin T\}$). Thus, we have $V(T)_T = \{z \in \mathbb{R}^{\#I} \mid z \in V(T)$ and $z_j = 0$ for all $j \notin T\}$.

Since the core of a quasi-balanced game is nonempty (Scarf 1967) and the NTU game derived from an exchange economy with convex preferences is quasi-balanced (Scarf 1971), we obtain nonemptyness of the core. Note that these considerations enable one to demonstrate the existence of core allocations without relying on existence of competitive equilibria and the fact that any competitive equilibrium allocation belongs to the core. Scarf (1967, 1971) calls the above condition balanced rather than quasi-balanced; however, it avoids confusion to use a separate

4 Hildenbrand and Kirman (1976, Chap. 3) is a good reference for economists who are unfamiliar with these concepts.

term for his original balancedness condition which was developed for NTU games arising from economies with quasiconcave utility functions. Since I emphasize concave utility functions, it is convenient to follow Billera's (1974) suggestion that the Scarf balancedness notion be termed "quasi-balanced." Convex combinations of allocations are the key to showing balancedness. Following Billera (1974) and Billera and Bixby (1974), define balancedness for NTU games as follows:

Definition 5.1 *A family $\mathcal{B}(S)$ of subsets of S is* balanced *(or a* balanced collection on *S) if there are nonnegative weights $w_T \geq 0$ for all $T \subseteq S$ such that $\sum_{\substack{T\subseteq S\\ T\ni i}} w_T = 1$ for all $i \in S$. For $S \subseteq I$, let $B(S)$ denote the set of all* balancing weights *for balanced collections on S; i.e.,*

$$B(S) = \Big\{ w\colon 2^S \to I\!R_+ \;\Big|\; \sum_{T\ni i} w_T = 1 \textit{ for all } i \in S \textit{ and } w_T = 0 \textit{ if } T \notin \mathcal{B}(S) \Big\}.$$

Definition 5.2 *The NTU game $V\colon 2^I \to I\!R^{\#I}$ is* balanced *if $V(I) \bigcup \big\{ \sum_{T\subseteq I} w_T V(T)_T \;\big|\; w \in B(I) \big\}$. Equivalently, it is* balanced *if $V(I) \supseteq \sum_{T\subseteq I} w_T V(T)_T$ for every $w \in B(I)$.*

Definition 5.3 *An NTU game is* totally balanced *if all of its subgames are balanced. In symbols, $V: 2^I \to I\!R^{\#I}$ is* totally balanced *if*

$$V(S) = \bigcup \Big\{ \sum_{T\subseteq S} w_T V(T)_T \;\Big|\; w \in B(S) \Big\} \textit{ for every } S \subseteq I,\, S \neq \emptyset.$$

Equivalently, it is totally balanced *if $V(S) \supseteq \sum_{T\subseteq S} w_T V(T)_T$ for every $w \in B(S)$ and every $S \subseteq I$ with $S \neq \emptyset$.*

The second version (in Billera and Bixby 1973; Mas-Colell 1975) of Definitions 5.2 and 5.3 facilitates the demonstration that concave utilities lead to (totally) balanced games. To see the equivalence, select a coalition S arbitrarily ($\emptyset \neq S \subseteq I$). Clearly S itself is a balanced collection with balancing weights $w_S = 1$ and $w_T = 0$ for all $T \neq S$. Hence $V(S) \supseteq \sum_{T\subseteq S} w_T V(T)_T$ implies $V(S) = \sum_{T\subseteq S} w_T V(T)_T$ for some choice of weights. Therefore $V(S) \supseteq \sum_{T\subseteq S} w_T V(T)_T$ for all $w \in B(S)$ implies $V(S) = \bigcup\big\{ \sum_{T\subseteq S} w_T V(T)_T \;\big|\; w \in B(S) \big\}$. Conversely, if $V(S) = \bigcup\big\{ \sum_{T\subseteq S} w_T V(T)_T \;\big|\; w \in B(S) \big\}$ then trivially $V(S) \supseteq \sum_{T\subseteq S} w_T V(T)_T$ for all $w \in B(S)$.

Definition 5.4 *An information sharing rule F is* nested *if for all $i \in I$ and all coalitions S and T with $\{i\} \subseteq S = \{s(1), \ldots, s(\#S)\} \subseteq T = \{t(1), \ldots, t(\#T)\} \subseteq I$, we have $f(S)^i\big(\mathcal{G}_{s(1)}, \ldots, \mathcal{G}_{s(\#S)}\big) \subseteq f(T)^i\big(\mathcal{G}_{t(1)}, \ldots, \mathcal{G}_{t(\#T)}\big)$ for any private information σ-fields $\mathcal{G}_1, \ldots, \mathcal{G}_{\#I}$ of the players in I.*

Theorem 5.5 *All NTU games with asymmetric information under the "X" information sharing rule are totally balanced if the "X" information sharing rule is nested.*

Corollary 5.6 *For the fine information sharing rule, every NTU game with asymmetric information is totally balanced.*

Corollary 5.7 *With private information sharing, every NTU game with asymmetric information is totally balanced.*

Remark 5.8 Maus (2003) has extended this analysis to a model without subjective probabilities, although his work requires that the set of states of the world be finite. He also argues (as in the original version of this paper) that there are counter-examples showing that information sharing rules (which he calls communication structures) that fail to be bounded (respectively, nested) give rise to NTU games that fail to be balanced (totally balanced).

6 Superadditivity

This section considers the effect of coalition information sharing rules on a basic property of cooperative games. While some interesting games arising from economics may violate superadditivity,[5] this feature is certainly an important one to examine for market games with asymmetric information. Superadditivity means that the addition of players to coalitions cannot lower or eliminate feasible payoffs for original coalition members. The NTU game with characteristic function given by the correspondence $V : 2^I \to I\!R^{\#I}$ is *superadditive* if for all $S \subseteq I$ and all $T \subseteq I$ with $S \cap T = \emptyset$, $V(S) \cap V(T) \subseteq V(S \cup T)$.

Propostion 6.1 *Superadditivity fails for coarse information sharing.*

Frankly, I am surprised that this observation – that addition of a player with no information and no endowment can lower payoffs – seems to be unknown in the literature. To the extent that one finds superadditivity to be an appealing property of a game, its lack castes doubt on the coarse core. An economic interpretation of superadditivity is the lack of (strict) diseconomies of scale and scope or the positivity of all externalities. These features are frequently assumed in microeconomic theory.

Remark 6.2 The superadditivity failure claimed in Proposition 6.1 for the coarse core can be demonstrated using the fact that a player with no information and no commodity endowment is not necessarily null because he can lower everyone's payoffs. By definition, for an NTU game $V : 2^I \to I\!R^{\#I}$, $i \in I$ is a *null player* if, for all coalitions $S \neq \emptyset$ with $i \notin S$,

$$V(S) \cap V\big(\{i\}\big) = V\big(S \cup \{i\}\big).$$

7 Cores with asymmetric information

To begin, recall the formal definitions. The *core* of an NTU game $V : 2^I \to I\!R^{\#I}$ is the set of all payoff vectors $(w_1, \ldots, w_{\#I}) \in I\!R^{\#I}$ such that $(w_1, \ldots, w_{\#I}) \in V(I)$

[5] For NTU games generated by economies with public goods, Guesnerie and Oddou (1979, 1981) examine some cases where superadditivity fails and analyze the relationships between superadditivity, balancedness, and nonemptiness of the core. Specifically, they show that lack of balancedness need not imply that the core is empty. While the public goods aspects of information might be expected to exhibit similarities to the economic model considered by Guesnerie and Oddou (1979, 1981), Remark 7.9 below demonstrates that the analogy does not hold.

(feasibility) and there does not exist a coalition $S \subseteq I$ and $(w'_i, \ldots, w'_{\#I}) \in V(S)$ such that $w_{i'} \geq w_i$ for all $i \in S$ with $w_{j'} > w_j$ for some $j \in S$ (coalition S cannot block). I examine the NTU case (without side payments) because it is the appropriate setting for pure exchange economies.

Definition 7.1 *The* core *of an economy with asymmetric information under the information sharing rule F consists of all state-dependent allocations* $(x_1, \ldots, x_{\#I})$ *where $x_i : \Omega \to I\!R^{\ell}_{+}$ for each $i \in I$ such that*

1. $\sum_{i \in I} x_i(\omega) = \sum_{i \in I} e_i$ *for (almost) all $\omega \in \Omega$,*
2. *each x_i is such that $x_i - e_i$ is $f(I)^i$-measurable, and*
3. *there does not exist a coalition $S(\emptyset \neq\subseteq I)$ and allocations $x_{i'} : \Omega \to I\!R^{\ell}_{+}$ for $i \in S$ such that $\sum_{i \in S} x_{i'}(\omega) = \sum_{i \in S} e_i$ for (almost) all $\omega \in \Omega$, each $x_{i'} - e_i$ is $f(S)^i$-measurable, and $EU_i(x_{i'}) \geq EU_i(x_i)$ for all $i \in S$ with $EU_j(x_{j'}) > EU_j(x_j)$ for some $j \in S$.*

This is the usual concept of the core of a (pure exchange) economy except for the informational constraints that each net trade defining a core allocation must be $f(I)^i$-measurable and that blocking must be accomplished via net trades that are $f(S)^i$-measurable for each member i of the blocking coalition S. Recall that my commodity space consists of (*ex ante*) state-dependent commodity bundles. Moreover, payoffs are given by the *ex ante* expected utilities associated with these state-dependent allocations.

The set of *core imputations* consists of those payoffs vectors $\big(EU_1\big(x_1(\cdot)\big), \ldots, EU_{\#I}\big(x_{\#I}(\cdot)\big)\big) \in I\!R^{\#I}$ where $(x_1, \ldots, x_{\#I}) : \Omega \to I\!R^{\#I\ell}_{+}$ belongs to the core. Note that the set of core imputations of an economy equals the core of the market game induced by the economy.

Three natural examples of cores with asymmetric information are the coarse core, the fine core, and the private information core – corresponding to the coarse, fine and private information sharing rules respectively. In section 5, the games derived from private and fine information sharing were shown to be totally balanced. This implies that their cores are nonempty.

Theorem 7.2 *The private information sharing rule core is nonempty.*

Remark 7.3 Yannelis (1991) provides a direct proof, based on Banach lattice methods, that exchange economies having essentially at most countably many states of the world necessarily have private core allocations. Allen (1992) also demonstrates that the private core is nonempty, using nestedness of the private information sharing rule combined with a simpler proof that the NTU game generated by a pure exchange economy with private information sharing is nonempty. Lefebvre (2001) provides yet another proof, based on fixed-point methods, which permits the underlying commodity space in each state of the world to be a Banach lattice.

Theorem 7.4 *The fine information sharing core is nonempty.*

Theorem 7.5 *If the information sharing rule is nested, then the core is nonempty.*

On the other hand, (quasi)-balancedness suffices for the nonemptiness of the core of an NTU game. For balancedness, the information available to various coalitions imposes a restriction only on the information of the grand coalition and not, in contrast to the case of totally balanced games, to all coalitions. While nestedness of information sharing rules implies balancedness of the associated games, a weaker condition is possible.

Definition 7.6 *An information sharing rule is* bounded *if for all*

$$(\mathcal{G}_1, \ldots, \mathcal{G}_{\#I}) \in \mathcal{F}^{**} \times \cdots \times \mathcal{F}^{**} \textit{ and every } j \in I,$$

$$f(I)^j(\mathcal{G}_1, \ldots, \mathcal{G}_{\#I}) \supseteq \sigma\Big(\bigcup_{S \subseteq I} f(S)^j(\mathcal{G}_{s(1)}, \ldots, \mathcal{G}_{s(\#S)}\Big) \textit{ where}$$

$$S = \{s(1), \ldots, s(\#S)\}.$$

Propostion 7.7 *Any nested information sharing rule is bounded. The converse is not true.*

Theorem 7.8 *Boundedness of the information sharing rule is a sufficient condition for balancedness. Hence, boundedness implies that the games have nonempty cores. In particular, if for all $j \in I$, $f(I)^j(\mathcal{G}_1, \ldots, \mathcal{G}_{\#I}) = \sigma\big(\bigcup_{i \in I} \mathcal{G}_i\big)$ or $f(I)^j = \mathcal{F}$, then the game is balanced and its core is nonempty.*

Remark 7.9 Under the coarse information sharing rule, NTU games with asymmetric information may fail to be balanced. To show explicitly that coarse information sharing actually can give an empty core, consider a two-state (heads, H, and tails, T, which each occur with probability one-half), two-good economy with three agents, of whom 1 and 2 are informed. Suppose that endowments are (1,1) for each trader, regardless of the state of the world, and let utilities be log linear and given by (writing x and y for allocations of the first and second good, respectively) the following:

$$u_1(x_1, y_1; H) = \frac{1}{3}\log x_1 + \frac{2}{3}\log y_1$$

$$u_1(x_1, y_1; T) = \frac{2}{3}\log x_1 + \frac{1}{3}\log y_1$$

$$u_2(x_2, y_2; H) = \frac{2}{3}\log x_2 + \frac{1}{3}\log y_2$$

$$u_2(x_2, y_2; T) = \frac{1}{3}\log x_2 + \frac{2}{3}\log y_2$$

$$u_3(x_3, y_3; H) = u_3(x_3, y_3; T) = \frac{1}{2}\log x_3 + \frac{1}{2}\log y_3$$

Then the only $\bigcap \mathcal{G}_i$-measurable allocation for the grand coalition which is a candidate for the core is $\big((1,1), (1,1), (1,1)\big)$ because it is the only constant function which is individually rational for each player. On the other hand, the coalition $\{1, 2\}$ can use their information $\{\Omega, \{H\}, \{T\}, \emptyset\}$ to block this by proposing the state-dependent allocations $\big((1-\epsilon, 1+\epsilon), (1+\epsilon, 1-\epsilon)\big)$ in state H and $\big((1+\delta, 1-\delta), (1-\delta, 1+\delta)\big)$ in state T for sufficiently small positive ϵ and δ.

(Observe that a similar example cannot be constructed with two agents because such games are automatically (totally) balanced even with coarse information sharing. Moreover, my counterexample defines a game which is not superadditive.)

Remark 7.10 If we modify the coarse information sharing rule by altering the information assigned to the grand coalition from
$f(I)(\mathcal{G}_1, \ldots, \mathcal{G}_{\#I}) = \left(\bigcap_{i\in I} \mathcal{G}_i, \ldots,\right.$
$\left.\bigcap_{i\in I} \mathcal{G}_i\right)$ to $f(I)(\mathcal{G}_1, \ldots, \mathcal{G}_{\#I}) = (\mathcal{G}_1, \ldots, \mathcal{G}_{\#I})$,
$f(I)(\mathcal{G}_1, \ldots, \mathcal{G}_{\#I}) = \left(\sigma\left(\bigcup_{i\in I} \mathcal{G}_i\right), \ldots,\right.$
$\left.\sigma\left(\bigcup_{i\in I} \mathcal{G}_i\right)\right)$, or $f(I)(\mathcal{G}_1, \ldots, \mathcal{G}_{\#I}) = (\mathcal{F}, \ldots, \mathcal{F})$, then boundedness is satisfied, the game is balanced, and its core is nonempty. This observation points out that only the information assigned to the grand coalition matters for balancedness and nonemptiness of the core.

Definition 7.11 *If $F = \{f(S) \mid S \subseteq I\}$ and $F' = \{f'(S) \mid S \subseteq I\}$ are information sharing rules for market games with asymmetric information with the same player set I and the same measurable space $(\Omega, \mathcal{F})$ of events, say that F* is stronger than *F' (written $F > F'$) if for all $S = \{s(1), \ldots, s(\#S)\} \subseteq I$, all $\left(\mathcal{G}_{s(1)}, \ldots, \mathcal{G}_{s(\#S)}\right) \in \mathcal{F}^{**} \times \cdots \times \mathcal{F}^{**}$, and all $j \in S$, we have $f(S)^j\left(\mathcal{G}_{s(1)}, \ldots, \mathcal{G}_{s(\#S)}\right) \supseteq f'(S)^j\left(\mathcal{G}_{s(1)}, \ldots, \mathcal{G}_{s(\#S)}\right)$. Say that F is comparable to F' (written $F \sim F'$) if for all $(\mathcal{G}_1, \ldots, \mathcal{G}_{\#I}) \in \mathcal{F}^{**} \times \cdots \times \mathcal{F}^{**}$, $f(I)(\mathcal{G}_1, \ldots, \mathcal{G}_{\#I}) = f'(I)(\mathcal{G}_1, \ldots, \mathcal{G}_{\#I})$.*

Remark 7.12 The relation $>$ is reflexive and transitive, so that "is stronger than" is indeed a preorder; it's obviously incomplete whenever $\mathcal{F}$ is nontrivial. The relation $\sim$ is an equivalence relation; "is comparable to" partitions information sharing rules into equivalence classes defined by identical information assigned to the grand coalition.

Theorem 7.13 *If $F \sim F'$ and $F > F'$, then the F core is contained in the F' core for any NTU game with asymmetric information.*

8 Other core concepts

Clearly there are additional explicit ways to define the core with asymmetric information, although our notion of information sharing rules encompasses all of the possibilities if the information of coalitions is taken as given rather than chosen strategically. Note, however, that the coarse core is not the most extreme one, as we could forbid all information use ($f(S)^i \equiv \{\Omega, \emptyset\}$). This leads to core allocations that are constant across states and that, in fact, equal precisely the core allocations in an economy without uncertainty having preferences representable by the unconditional expected utilities of the asymmetric information economy. At the other extreme, we could remove the asymmetric information by requiring pooling of all information ($f(S)^i = \sigma\left(\bigcup_{j\in I} \mathcal{G}_j\right)$). Then all coalitions have access to the same information and we should obtain a version of the core with contingent commodities and possibly limited transfers across states – the analogue of incomplete markets for a model with no markets whatsoever. Additional alternatives include complete information ($f(S)^i \equiv \mathcal{F}$) and common information ($f(S)^i = \bigcap_{j\in I} \mathcal{G}_j$).

All of these information sharing rules are symmetric and yield totally balanced games, although the cores cannot be compared by Theorem 7.13.

Implicitly I have already suggested several criteria to consider in choosing among information sharing rules and their associated core concepts. If we wish to retain nonemptiness of the core, then we should select an information sharing rule that leads to a balanced or totally balanced game.

Here are some additional possible criteria for cores with asymmetric information:

1. *Core equivalence* – not good, because this should be demonstrated rather than taken as an axiom; also, the choice of an appropriate competitive equilibrium concept with asymmetric information is problematic (see section 11).
2. *Characterization of blocking coalitions* ("small and similar," see Grodal 1972).
3. *Number of blocking coalitions* (asymptotically one-half) – fails for both the coarse and fine cores if the noncore allocation requires information from two types of agents; also fails for the private information core because two agents with the same information are required (compare to Mas-Colell 1978; Shitovitz 1983).
4. *Equal treatment property* in replica economies – my conjecture is that this fails for the private information and fine cores, but holds for the coarse core.

9 Wilson's cores

My results on the coarse and fine cores might seem to contradict the conclusions of Wilson (1978). In particular, his "fine" core (which I term the full communication structure core) can be empty, whereas my fine information sharing rule yields a totally balanced game which therefore has a nonempty core. To the contrary, he argues that his "coarse" core (which I term the null communication structure core) is nonempty due to the balancedness of a related game. The apparent contradiction can be explained by the fact that my definitions of these core concepts – and, more fundamentally, of the notions of blocking and of the utilization of information by coalitions—are basically different. In particular, Wilson (1978) analyzes *ex post* blocking in some particular event that a coalition can distinguish, while I consider *ex ante* blocking via a state-dependent allocation that gives higher expected utility to coalition members and that is measurable with respect to the coalition's information as well as feasible for the coalition in the usual sense. Moreover, Wilson's (1978) communication structures differ from my information sharing rules in that the communication structures define which information is permitted – but not required – to be shared among coalition members. Given a nonnull communication structure, a coalition can choose the information that it wishes to have available to its members. In particular, insurance can be created by the deliberate choice to commit to disregard some information that could be communicated. The endogeneity of this information choice alters the characteristic function games.

The null communication structure forbids players from sharing information, so that everyone simply keeps his own private information. However, the feasibility requirement for state-dependent allocations is based on the pooled information of all players. In terms of my notation for information sharing rules, the null communication structure is defined by the following conditions for all $i \in I$:

1. $f\big(\{i\}\big)(\mathcal{G}_i) = \mathcal{G}_i$,
2. $f(S)^i\big(\mathcal{G}_{s(1)}, \ldots, \mathcal{G}_{s(\#S)}\big) = \bigcap_{j\in S} \mathcal{G}_j$ for $S \neq I$, $S \ni i$, and
3. $f(I)^i(\mathcal{G}_i, \ldots, \mathcal{G}_{\#I}) = \sigma\big(\bigcup_{i\in I} \mathcal{G}_i\big)$.

Notice that the grand coalition receives special treatment; feasibility imposes no additional measurability requirements beyond the condition for singletons that $x_i(\cdot)$ be $\mathcal{G}_i$-measurable (for all $i \in I$). Thus, my Remark 7.9 on the coarse core is not relevant. Instead, the null communication structure yields a unique bounded information sharing rule, so that the null communication structure core is nonempty. However, Wilson (1978) uses a different method to show balancedness because of his *ex post* blocking concept. He defines an artificial game having admissible player-event pairs as its players and shows that this related game is balanced.[6]

The full communication structure permits coalitions $S \subseteq I$ to choose their information sub-σ-fields $\mathcal{H}_{S,i}$ endogenously as part of their strategies. This constitutes a major departure from my fine information sharing rule in which each member of coalition S has exactly the information $\sigma\big(\bigcup_{i\in S} \mathcal{G}_i\big)$. In fact, Wilson (1978) requires the following two conditions on the (binding agreements) $\mathcal{H}_{S,i}$ under the full communication structure:

1. $\mathcal{H}_{\{i\},i} = \mathcal{G}_i$ for all $i \in I$, and
2. $\mathcal{H}_{S,i}$ (for $i \in S$, $\#S > 1$) satisfies $\mathcal{G}_i \subseteq \mathcal{H}_{S,i} \subseteq \sigma(\bigcup_{i\in S} \mathcal{G}_i)$.

The endogeneity of coalitions' choices $\mathcal{H}_{S,i}$ subject to (b) implies that one cannot appeal to Scarf's (1967) balancedness criterion. When the grand coalition picks sufficiently small sub-σ-fields $\mathcal{H}_{I,i}$, convex combinations of (for singletons) $\mathcal{G}_i$-measurable or (more generally) $\mathcal{H}_{S,i}$-measurable allocations can fail to be measurable, so that the state-dependent allocations violate feasibility (for the grand coalition). Similarly, the market games literature cannot easily be used here because the consumption sets depend on the coalitions and their strategic information choices. Since such endogenous information decisions do not form a convex (or compact) subset of a topological vector space, the Billera and Bixby (1974) proof cannot apparently be modified to incorporate such enlarged strategies. Wilson (1978) establishes that the full communication structure core can be empty by exhibiting an example in which the grand coalition can block any candidate core allocation. The counterexample involves potential risk sharing among traders and the creation by the grand coalition of "new markets" for insurance contracts. Indeed, the Pareto optimality concept implied by this example's feasibility definition is stronger than the informational efficiency standard in the literature in that the planner can implicitly implement better (less risky) allocations by opening markets and making insurance contracts available by committing not to communicate information. Essentially, the main idea is that the grand coalition can make binding agreements to burn information; when information is destroyed, risks can be insured.

More generally, the possibility for information sharing is included in the strategy spaces for Wilson's (1978) nonnull communication structures. As soon as information is endogenized in this way, one can construct examples in which balancedness fails because the grand coalition chooses to commit to ignore some

[6] Kobayashi (1980) could not extend this proof strategy to the case of countable partitions because the induced artificial game would need infinitely many players.

information. The resulting reduced game (or second stage of the implicit game in which coalitions first choose information, then allocations) having only allocations as strategies therefore fails to be balanced. While endogeneity of information is desirable, this approach has the disadvantage that a player or coalition may wish to renege on a coalition's commitment to ignore or destroy information. The potential for such unauthorized *ex post* public announcement of private information creates incentive problems unless one were to restrict consideration to information sharing strategies which are, in some sense, self enforcing (which need not lead to balancedness).

10 Incentive compatibility

Building on Wilson's (1978) pathbreaking article as well as the huge literature on the theory of incentives and mechanism design, a large and still growing body of research focuses on incentive compatibility and the core of an economy with asymmetric information. For example, see the survey of this area by Forges, Minelli and Vohra (2002) and the introductory comments in Allen and Yannelis (2001) and Glycopantis and Yannelis (2005), as well as the references listed in these three pieces. Space constraints and avoidance of duplication prevent me from exhaustively summarizing all of the results in this burgeoning literature.

Consider first of all the issue of whether any of the cores defined by information sharing rules satisfy incentive compatibility for all core allocations. An obviously weaker form of the question is to ask for incentive compatibility for at least one core allocation, where the core is defined by some particular information sharing rule for which the core is nonempty. Both forms of the question require one to specify the meaning of incentive compatibility in an economy with asymmetric information. Koutsougeras and Yannelis (1993) examine the coarse, private, and fine core (which may be empty) and show that all satisfy coalitional incentive compatibility as defined in Krasa and Yannelis (1994).

In defining incentive compatibility for situations with more than one player or economic agent, two basic decisions must be made. The first concerns the timing of commitment to contracts versus the timing of information revelation. Do agents commit to entire state-dependent allocations before learning $\mathcal{G}_i$? An affirmative answer leads to *ex ante core* and *ex ante incentive compatibility* concepts.[7] If, to the contrary, agents can wait until observing a realization of the information

[7] The *ex ante* individually incentive compatible core has been examined by Allen (1991c, 1995, 1999, 2003), Forges, Mertens and Vohra (2002), Forges and Minelli (2001), and Vohra (1999), among others. Allen (1991c, 2003) shows, in a complicated but robust example, that the *ex ante* incentive compatible core of the NTU game generated by a pure exchange economy can be empty; Vohra (2002) also provides an example of an empty *ex ante* incentive compatible core. Various randomization devices have been explored in order to understand whether the resulting convexification can alleviate the problem. Allen (1991c, 2003) shows that weakening the market clearing almost surely requirement to market clearing on average can indeed restore nonemptyness, albeit admittedly at the expense of a definition that is economically somewhat unconvincing. However, the contribution of the Allen (1991c, 2003) article should certainly be viewed as the provision of an explicit counterexample – a negative result – and not a positive statement that randomization can solve the problem. Forges, Mertens and Vohra (2002) prove that although the *ex ante* incentive compatible core can be empty, but with quasilinearity (i.e., in games with side payments) it is generically nonempty. Replications of such economies have been shown to yield more positive results for the *ex ante* incentive compatible core; see Allen

one obtains the *interim core* and *interim incentive compatibility*.[8] Finally, one can consider the *ex post* core. [9] Note, however, that *ex post incentive compatibility* isn't interesting. Incentive compatibility at the *interim* stage should be based on the information available to traders at that point and conditions should be added to ensure that coalitions (including the grand coalition) can detect incentive compatibility violations. Thus, one has the second basic decision: should incentive compatibility be imposed on the individual level (independent of the coalition) or should an agent's incentive compatibility constraints depend on the coalition?[10] Yet another possibility – as of now, apparently unexplored – is to impose a network structure (either to define allowable coalitions and hence cores with restricted coalition structures or, on each coalition in order to define pairwise communication possibilities within coalitions) and then to require that all messages be incentive compatible. Also related to implementation is the interesting issue of endogenous enforcement.[11]

11 Other properties

This section explores the beginnings of a literature that focuses on some additional properties of core allocations with asymmetric information. For the results comparing cores and competitive equilibria, finiteness of the set of states of the world is assumed. Hervés-Beloso, Moreno-García, and Yannelis (2005) provide a version of the first and second welfare theorems for a finite pure exchange economy.[12] For core equivalence and convergence results, see Einy, Moreno, and Shitovitz (2000a,b, 2001), Serrano, Vohra, and Volij (2001), and Forges, Minelli, and Vohra (2001). Learning in dynamic models is considered by Koutsougeras and Yannelis (1999) and Serfes (2001).

(1995) and Forges, Heifetz, and Minelli (2001). Finally, in a large economy with an atomless continuum of agents, a dispersion condition on the distribution of initial endowments suffices to guarantee that the *ex ante* incentive compatible core is nonempty (see Allen 1999).

[8] With *interim* incentive compatibility, the incentive compatible coarse core of Wilson (1978) (recall that his definition features pooled information for the grand coalition, so that his null communication structure information sharing rule is bounded) is nonempty. Ichiishi and Idzik (1996) examine incentive compatibility for the *interim* core by imposing balancedness on the resulting NTU games using the Harsanyi (1967–68) formulation of games with incomplete information as adapted to cooperative games by Myerson (1984). [Forges, Mertens and Vohra (2002), Forges and Minelli (2001), Vohra (1999), and Forges, Heifetz and Minelli (2001) all use this framework to study the *ex ante* incentive compatible core (see also Forges, Minelli and Vohra 2002).]

[9] Hahn and Yannelis (1997) systematically examine the relationships among *ex ante*, *interim*, and *ex post* core concepts using private, common, and pooled information sharing. See also the discussion of coalitional incentive compatibility in Glycopantis and Yannelis (2005).

[10] The latter case leads to a variety of possible definitions of group or coalitional incentive compatibility in the more recent literature.

[11] See Koeppl (2002, 2004), Koeppl and MacGee (2001), and Krasa and Villamil (2000).

[12] Extension to a large economy with a continuum of traders should be straightforward under appropriate measurability and integrability conditions, but extension to infinitely many states of the world apparently fails due to the fact that the (exact) Liapounov Theorem doesn't hold in infinite dimensional spaces.

12 Perspective

Most of the literature discussed in Sections 10 and 11 uses the assumption that the set of states of the world is finite. (One can obviously augment this with an additional set of states that all agents agree occurs with total probability zero. Moreover, for some results, the finite set of states of the world can be replaced by an infinite but countable discrete set of states, each of which occurs with strictly positive probability.) This finiteness restriction arises in studies of incentive compatibility with asymmetric information partly because, although extensions to more general probability spaces are possible, most of the literature on games with incomplete information customarily assumes that type spaces are finite. In addition, it is difficult to envision infinitely many incentive compatibility constraints. For comparisons to rational expectations equilibria, finiteness clearly permits one to focus on the fully revealing case. Yet this observation also causes me to suspect that this case is misleading, not only for cores but also for economies or games with asymmetric information in general.

In addition, I believe that it is important for research about asymmetric information to permit very general probability spaces – i.e., abstract probability triples $(\Omega, \mathcal{F}, \mu)$ where Ω is an arbitrary abstract set of states of the world, $\mathcal{F}$ is an arbitrary σ-field of measurable subsets of Ω, and μ is an arbitrary (countably additive) probability measure defined in $(\Omega, \mathcal{F})$. We should only use measure-theoretic properties and not topological or vector space properties, because axiomatic treatments of decision theory under uncertainty yield only a probability space. Different agents should ideally be permitted to have different subjective probabilities, possibly shared within coalitions. (Alternatively, one could represent subjective probabilities by sets of possible probabilities on $(\Omega, \mathcal{F})$ and consider a maximin criterion which also incorporates the agents' relevant information sub-σ-field.)

In the literature on cores (and also other cooperative solution concepts) with asymmetric information, there are three main research strategies for the specification of how the feasible sets of decisions for different coalitions relate to each other. One approach – the one taken here when general information sharing rules are defined – is to begin with a very general model, establish its basic properties, and then attempt to characterize broad classes of cases for which some particular more specific results hold. The opposite extreme is to focus on a small number of especially interesting or important information sharing rules. Yet within this approach, one sees two distinct implicit criteria for prioritizing various information sharing rules according to their potential economic interest or importance. One can insist that all coalitions be treated symmetrically. An opposite extreme, which also arises frequently in the literature, is to endow the grand coalition with distinct powers not held by other coalitions. The device obviously facilitates the demonstration that some core is nonempty. Similarly one can define incentive compatibility constraints symmetrically for all coalitions. My personal opinion is that, for most purposes in economics, the grand coalition should not be endowed with special powers (unless, of course, the special powers arise endogenously in some appropriate fully-specified model).

Yet another general modeling issue for cores with asymmetric information concerns whether free disposal should be permitted. An argument against free disposability is that it may convey additional information when agents observe the state-dependent aggregate disposal vector. Yet the elimination of free disposal can impose additional information-measurability conditions on state-dependent individual net trade vectors in different coalitions – specifically, an agent's state-dependent net trade must be measurable not only with respect to the agent's own information in the coalition but also with respect to the pooling of the information that is available to each other agent in the coalition. [This leads to the concept of publicly predictable information sharing rules; see Allen (1993).] When free disposal is permitted, what can be observed in coalitions? Analogous issues arise when initial endowments may be state dependent. In the analysis presented in this paper, I do allow blocking to involve free disposal, but I do not permit free disposal in the definition of feasibility for the grand coalition. This contradicts the desideratum elucidated above that the grand coalition should not be treated differently. I justify this modelling choice by the argument that blocking should be viewed as hypothetical. Coalitions don't actually engage in blocking in the core – they don't secede from the grand coalition – but instead they use blocking arguments as a group to prevent allocations that can be improved upon by the coalition. However, at an actual core allocation, any agent could see the occurrence of disposal and thereby possibly gain additional information about the state of the world. Hence, free disposal should be prohibited in the feasibility definition for the grand coalition, at least as a first step.

As research develops on cores with asymmetric information, the information available to members of various coalitions is becoming increasingly endogenized in our models. For example, compare the private core to coalitionally incentive compatible cores. This suggests that perhaps we should attempt to "price" the information as part of our solution concepts, by permitting agents to trade information for goods (see Allen 1986a,b). Doing so would require careful consideration of the familiar free rider problem, or equivalently the Grossman and Stiglitz (1976) dichotomy.

Additional open research challenges that arise in the study of cores with asymmetric information include considerations of computational complexity limitations, computational costs, the implicit sizes of message spaces, and costs of and/or constraints on information transmission. These issues seem to become especially important when incentive compatibility and especially coalitional incentive compatibility are imposed.

One could easily provide a long list of open questions about extensions of the basic models discussed in this paper to include more complicated general features that are not inherent to coalitions with asymmetric information. In addition, one could elucidate many versions of other cooperative game-theoretic solution concepts, such as the value. While it could be worthwhile to investigate some of these variations carefully, judgment is required to prioritize the enormous collection of research topics that can arise in this area. Instead of investigating many of them piecemeal, I am convinced that systematic treatment of general classes of properties is more likely to be illuminating for economic theory. For instance, when we attempt to establish nonemptyness of cores with asymmetric information, we find that proofs can be divided into three main classes: those that follow the tack taken here of deriving cooperative games and checking for balancedness, those that directly establish the existence of allocations in the core (e.g., Yannelis 1991), and those that argue (as one would when teaching the core of an economy without uncertainty) that there are competitive equilibrium allocations which must necessarily also be core allocations (as in Allen 1999). In this work, the structure of the proof not only suggests some useful assumptions but also helps one to understand the nature of the difficulties that arise when information feasibility conditions depend on coalitions (how can one define an appropriate analogue of competitive equilibrium when the commodity space depends on one's coalition?) and when incentive compatibility leads to nonconvex sets of feasible allocations. A different but potentially valuable approach is to focus effort on particular topics where one expects, paradoxically, to be surprised.

Finally, I am convinced that one should and must focus first on finding appropriate ways to define basic concepts, such as information sharing rules or coalitional incentive compatibility notions. Then, if appropriate definitions lead robustly/generically to empty or badly behaved cores, one should accept those conclusions and remember that the core is not necessarily an ideal solution concepts.

Appendix: Proof of Theorem 4.1

By definition, we have $V(\emptyset) = \{0\}$. Moreover, the fact that the inequalities defining $V(S)$ apply only for $i \in S$ implies that $V(S)$ is a cylinder set. It's comprehensive also from the "less than or equal to" requirement in its definition. Similarly, convexity of the $V(S)$ follows from concavity of the $u_i(\cdot, \omega)$ on $I\!R^{\ell}_+$.

It remains to prove that the sets $V(S)$ are closed subsets of $I\!R^{\#I}$. Let $w^n = (w^n_1, \ldots, w^n_{\#I}) \in V(S)$ for all n with $\lim_{n\to\infty} w^n = w \in I\!R^{\#I}$. I need to prove that $w \in V(S)$. By the definition of $V(S)$, for $i \in S$ and all $n = 1, 2, \ldots$, there exist functions $z^n_i \colon \Omega \to I\!R^{\ell}$ such that $Z^n_i(\cdot)$ is $f(S)^i$-measurable, $Z^n_i(\omega) + e_i(\omega) \in I\!R^{\ell}_+$ for all $\omega \in \Omega$ and all $i \in S$, $\sum_{i \in S} z^n_i(\omega) = 0$ for almost all $\omega \in \Omega$, and $w^n_i \leq \int_\Omega u_i\big(z^n_i(\omega) + e_i(\omega); \omega\big)\, d\mu(\omega)$ for $i \in S$. By assumption, the $u_i(\cdot; \omega)$ are uniformly bounded above and below on compact subsets of $I\!R^{\ell}_+$.

It's a consequence of Dunford-Pettis (1940; see Dunford and Schwartz 1958, Exercise IV.13.68, pp. 349–350, and Exercise IV.13.25, p. 342) that the uniformly bounded sequence $z^n(\cdot) = \big(z^n_{s(1)}(\cdot), \ldots, z^n_{s(\#S)}(\cdot)\big) \colon \Omega \to I\!R^{\#S\ell}$ contains a weakly convergent subsequence $z^{n'}(\cdot) = \big(z^{n'}_{s(1)}(\cdot), \ldots, z^{n'}_{s(\#S)}(\cdot)\big) \colon \Omega \to I\!R^{\#S\ell}$. Let $\bar{z}(\cdot) = \big(\bar{z}_{s(1)}(\cdot), \ldots, \bar{z}_{s(\#S)}(\cdot)\big) \colon \Omega \to I\!R^{\#S\ell}$ denote its limit. For all n, I have $z^n \in \Big\{ z = (z_{s(1)}(\cdot), \ldots, z_{s(\#S)}(\cdot) \colon \Omega \to I\!R^{\#S\ell} \;\Big|\; \sum_{i \in S} z_i(\omega) = 0$ for almost every $\omega \in \Omega$ and, for all $i \in S$, $z_i(\cdot)$ is $f(S)^i$-measurable and $z_i(\omega) + e_i(\omega) \in I\!R^{\ell}_+$ for all $\omega \in \Omega \Big\} = M(S)$, which is strongly (i.e., for the $\mathcal{L}^1$ norm topology) closed and convex.[13] By Theorem 3.12 of Rudin (1973, p.64), it's also weakly closed so that $\sum_{i \in S} \bar{z}_i(\omega) = 0$ for almost every $\omega \in \Omega$ and, for all $i \in S$, $\bar{z}_i(\cdot)$ is $f(S)^i$-measurable and $\bar{z}_i(\omega) + e_i(\omega) \in I\!R^{\ell}_+$ for all $\omega \in \Omega$. Define a sequence $\{z^{n',n''}(\cdot)\}_{n''}$ of sequences recursively by deleting the first term so that $z^{n',1}(\cdot) = z^{n'+1}(\cdot)$, $z^{n',2}(\cdot) = z^{n'+2}(\cdot), \ldots, z^{n',n''}(\cdot) = z^{n'+n''}(\cdot)$ and note that for all n'', $z^{n'n''}(\cdot)$ converges weakly to $\bar{z}(\cdot)$ as $n' \to \infty$. By Theorem 3.13 of Rudin (1973, p. 65) or Corollary V.3.14 in Dunford and Schwartz (1958, p. 422), for each n'' there are convex combi-

[13] More precisely, there are representatives of the $\mathcal{L}^1(\Omega, \mathcal{F}, \mu; I\!R^{\#S\ell})$ equivalence classes in $M(S)$ that satisfy $z_i(\omega) + e_i(\omega) \in I\!R^{\ell}_+$ for all $\omega \in \Omega$.

nations of finitely many terms in the sequence that converge strongly to $\bar{z}(\cdot)$. In terms of my notation, for each n'' there exist $\alpha_m^{n',n''} \in [0, 1]$ with $\sum_{n'} \alpha_m^{n',n''} = 1$ such that the sequence $\sum_{n'} \alpha_m^{n',n''} z^{n',n''}(\cdot)$ converges strongly to $\bar{z}(\cdot)$ as $m \to \infty$, where only finitely many $\alpha_m^{n',n''}$ terms in each summation are nonzero. For each n'', take a subsequence if necessary so that $\left\| \sum_{n'} \alpha_{m'}^{n',n''} z^{n',n''}(\cdot) - \bar{z}(\cdot) \right\|_1 \leq \frac{1}{m'}$ for all m'. Now diagonalize and consider the sequence $\sum_{n'} \alpha_{m'}^{n',n'} z^{n',n'}(\cdot)$. It can be written as $\sum_{n' \geq m'} \alpha_{m'}^{n'} z^{n'}(\cdot)$ because the diagonalization process guarantees that its m'-th term does not attach positive weights to $z^{n'}(\cdot)$ terms with $n' < m'$. Because $\left\| \sum_{n' \geq m'} \alpha_{m'}^{n'} z^{n'}(\cdot) - \bar{z}(\cdot) \right\|_1 \leq \frac{1}{m'}$, the result $\sum_{n' \geq m'} \alpha_{m'}^{n'} z^{n'}(\cdot)$ of the diagonalization converges strongly to $\bar{z}(\cdot)$ as $m' \to \infty$. By Theorem 2.5.1 of Ash (1972, p. 92), $\sum_{n' \geq m'} \alpha_{m'}^{n'} z^{n'}(\cdot)$ also converges to $\bar{z}(\cdot)$ in probability and I can use the extended dominated convergence theorem (see Ash 1972, p. 96) upon noting that uniform boundedness of the $z^{n'}(\cdot)$implies that $\sum_{n' \geq m'} \alpha_{m'}^{n'} z^{n'}(\cdot)$ is bounded also. Hence, because the $u_i(\cdot; \omega)$ are uniformly bounded on compact subsets of $I\!R_+^\ell$,

$$
\begin{aligned}
&\int_\Omega u_i(\bar{z}_i(\omega) + e_i(\omega); \omega)\, d\mu(\omega) \\
&\quad = \int_\Omega u_i\left(\left(\lim_{m' \to \infty} \sum_{n' \geq m'} \alpha_{m'}^{n'} z_i^{n'}(\omega) \right) + e_i(\omega); \omega \right) d\mu(\omega) \\
&\quad = \int_\Omega \lim_{m' \to \infty} u_i\left(\left(\sum_{n' \geq m'} \alpha_{m'}^{n'} z_i^{n'}(\omega) \right) + e_i(\omega); \omega \right) d\mu(\omega) \\
&\quad = \lim_{m' \to \infty} \int_\Omega u_i\left(\left(\sum_{n' \geq m'} \alpha_{m'}^{n'} \left(z_i^{n'}(\omega) + e_i(\omega) \right) \right); \omega \right) d\mu(\omega) \\
&\quad \geq \lim_{m' \to \infty} \int_\Omega \left(\sum_{n' \geq m'} \alpha_{m'}^{n'} u_i(z_i^{n'}(\omega) + e_i(\omega); \omega) \right) d\mu(\omega) \\
&\quad = \lim_{m' \to \infty} \sum_{n' \geq m'} \alpha_{m'}^{n'} \int_\Omega u_i(z_i^{n'}(\omega) + e_i(\omega); \omega)\, d\mu(\omega) \\
&\quad \geq \lim_{m' \to \infty} \sum_{n' \geq m'} \alpha_{m'}^{n'} w_i^{n'} = w_i,
\end{aligned}
$$

where the first inequality follows from Jensen's Inequality and concavity of the $u_i(\cdot; \omega)$ on $I\!R_+^\ell$). Therefore I have exhibited, for $i \in S$, $f(S)^i$-measurable functions $\bar{z}_i : \Omega \to I\!R^\ell$ which satisfy $\sum_{i \in S} \bar{z}_i(\omega) = 0$ for almost all $\omega \in \Omega$, $\bar{z}_i(\omega) + e_i(\omega) \in I\!R_+^\ell$ for all $i \in S$ and all $\omega \in \Omega$, and $w_i \leq \int_\Omega u_i(\bar{z}_i(\omega) + e_i(\omega); \omega)\, d\mu(\omega)$ for all $i \in S$. This proves that $(w_1, \ldots, w_{\#I}) \in V(S)$, as desired. □

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